K-5 Mathematics Standards Review

The goal for revising the Idaho Standards is to strengthen the clarity without jeopardizing the coherence and progression of ideas in the original Common Core documents. The following document is a sample of ways in which I believe our Idaho standards could be modified to meet the needs of our students.

Number Sense

Number sense is a fundamental bridge to algebraic thinking for middle and high school mathematics. Students with strong number sense understand numbers, ways to represent numbers, relationships among numbers, and number systems. They make reasonable estimates, compute fluently, use reasoning strategies (relate operations, such as addition and subtraction, to each other), and use visual models based on their number sense to solve problems. Students who never develop strong number sense will struggle with nearly all mathematical domains, from measurement and geometry to data and equations.

Many of the samples in this document are possible adjustments that could be integrated to draw out the number sense already included in the standards but not explicitly stated. For example, the language of Idaho standards doesn't always explicitly draw attention to the connection between place value and number sense. Therefore, I have included language from Florida B.E.S.T. standards to draw attention to the necessary connections which I hope will provide clarity for the teacher's diagnosis of a student's level of number sense.

Mastery Standards

The sample document purposefully draws attention to fluency and developmental milestone standards in each grade level. A separate heading labeled "mastery standards" has been included at the end of each grade level. These standards have been extracted from the Florida B.E.S.T. appendix, however, a collaborative effort from the Idaho committee members and leaders will be needed to ensure coherence across the grade levels.

Multiple Methods/Strategies

Many of our standards use language that expresses the need for students to use multiple representations and methods to solve a problem. Please see the standards that have been boxed throughout the document and the grade 5 example below:

> 5.NBT.5 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

The teacher provides multiple opportunities for students to make sense of the strategies at the instructional level. As these strategies are repeatedly used in ways that make sense to the students, they begin to understand and internalize the relationships that exist between and among numbers. This leads to fluency. I believe there have been professional misunderstandings around this language which has led to a high level of frustration from

parents, teachers and students. The end goal is for a student to solve the most efficient way possible. Explicit instruction of strategies which focus on students understanding the connections between approaches is critical. Students should be familiar with multiple strategies but should be able to select and use the strategy with which they most closely connect and understand, with the ultimate of goal of supporting students to use more efficient strategies. Sending homework or asking a student to complete the three ways as an evaluation of mastery is a flawed instructional strategy, not a flaw in the standard. The work of each Idaho committee member must focus on each of these standards to provide clarity and reduce the confusion.

Prioritization of Standards

Not all content in a given grade is emphasized equally in the standards. Some domains require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Idaho Standards for Mathematical Practice. The Mastery Standards should be included in the “greater emphasis” category. Students should spend the large majority of their time on the major work of the grade. Supporting work and, where appropriate, additional work can engage students in the major work of the grade. The major work of the grade level standards is also aligned with the Blueprint for our Smarter Balanced Assessment for grades 3-5. The allocations of emphasis can be found throughout Massachusetts, Kansas, Louisiana, North Carolina, and many other state’s standard documents.

Kindergarten Content Standards

Counting and Cardinality K.CC

A. Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Starting at a given number, count forward within 100 and backward within 20.
   Clarification: When counting forward by ones, students are to say the number names in the standard order and understand that each successive number refers to a quantity that is one larger. When counting backward, students are to understand that each succeeding number in the count sequence refers to a quantity that is one less.
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

B. Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger. Recognize the one more pattern of counting using objects.
   d. Given a verbal or written number from 0 to 20, count out that many objects.
5. Given a group of up to 20 objects, count the number of objects in that group and represent the number of objects with a written numeral. State the number of objects in a rearrangement of that group without recounting.

C. Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for groups with up to 10 objects, e.g., by using matching and counting strategies.
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings,[1] sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

[1] Drawings need not show details, but should show the mathematics in the problem.

4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MASTERY STANDARD:

5. Fluently add and subtract within 5, including zero.
   Clarification: Students are fluent when they display accuracy (correct answer), efficiency (in about 3-5 seconds without resorting to counting), and flexibility.

Mastery Standards **K.MS**

1. Fluently add and subtract within 5, including zero.
2. *Clarification: Students are fluent when they display accuracy (correct answer), efficiency (in about 3-5 seconds without resorting to counting), and flexibility.(K.OA.5)*
3. Count to 100 by ones and by tens. (K.CC.1)
4. Locate, order and compare whole numbers up to 20. (K.CC.3; K.CC.7)

Grade 1 Content Standards

Operations and Algebraic Thinking 1.OA

A. Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations (number sentences) with a symbol for the unknown number to represent the problem.[2]
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

B. Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations to add. [3]

   For example, when adding numbers order does not matter. If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative property of addition). When adding zero to a number, the result is the same number (Identity property of zero for addition).
4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

C. Add and subtract within 20.

1. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
2. Add and subtract within 20. Use mental strategies such as counting on; making 10 (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a 10 (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

   Clarification: As these strategies are repeatedly used in ways that make sense to the students, they begin to understand and internalize the relationships that exist between and among numbers.

   a. MASTERY STANDARD: Recall addition facts with sums to 10 and related subtraction facts with automaticity.

   Clarification: Students are fluent when they display accuracy (correct answer), efficiency (in about 3-5 seconds without resorting to counting), and flexibility.

D. Work with addition and subtraction equations.

3. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

   For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
4. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

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[2] See Glossary, Table 1.

[3] Students need not use formal terms for these properties.

For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.

Number and Operations in Base Ten 1.NBT

A. Extend the counting sequence.

1. Starting at a given number, count forward and backwards within 120 by ones. Skip count by 2s to 20 and by 5s to 100. In this range, read and write numerals using standard form and expanded form and represent a number of objects with a written numeral.
   Example: The number seventy-five written in standard form is 75 and in expanded form is 70 + 5.

B. Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Compose and decompose two-digit numbers in multiple ways using tens and ones.

   Example: The number 37 can be expressed as 3 *tens* + 7 *ones*, 2 *tens* + 17 *ones* or as 37 *ones*.
   Understand the following as special cases:
   - **a.** 10 can be thought of as a bundle of ten ones—called a "ten."
   - **b.** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   - **c.** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

C. Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. Identify arithmetic patterns of 10 more and 10 less than using strategies based on place value.
6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Mastery Standards 1.MS

1. Recall addition facts with sums to 10 and related subtraction facts with automaticity (1.OA.6a).

   Clarification: Students are fluent when they display accuracy (correct answer), efficiency (in about 3-5 seconds without resorting to counting), and flexibility.
2. Starting at a given number, count forward and backwards within 120 by ones. Skip count by 2s to 20 and by 5s to 100 (1.NBT.1).
3. Order and compare whole numbers up to 100. (1.NBT.1; 1.NBT.3)

Grade 2 Content Standards

Operations and Algebraic Thinking 2.OA

A. Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[4]

B. Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[5] By end of grade 2, know from memory all sums of two single-digit numbers and related differences.
   For example, the sum 6 + 5 = 11 has related differences of 11 – 5 = 6 and 11 – 6 = 5.

   a. MASTERY STANDARD--Recall addition facts with sums to 20 and related subtraction facts with automaticity.
   Clarification: Students are fluent when they display accuracy (correct answer), efficiency (in about 3-5 seconds without resorting to counting), and flexibility.

C. Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to five rows and up to five columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten 2.NBT

A. Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens—called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. MASTERY STANDARD--Count within 1,000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting starting at any number.
3. Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form.
4. MASTERY STANDARD--Order and compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

   Example: The numbers 424, 178 and 475 can be arranged in ascending order as 178, 424 and 475.

B. Use place value understanding and properties of operations to add and subtract.

5. MASTERY STANDARD--Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

[4] See Glossary, Table 1.
[5] Strategies such as counting on; making tens; decomposing a number; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Use mental strategies to add or subtract a number that is ten more, ten less, one hundred more and one hundred less than a given three-digit number.

   Example: The number 236 is one hundred more than 136 because both numbers have the same digit in the ones and tens place, but differ in the hundreds place by one.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[6]

Mastery Standards 2.MS

1. Recall addition facts with sums to 20 and related subtraction facts with automaticity. (2.OA.2a). *Clarification: Students are fluent when they display accuracy (correct answer), efficiency (in about 3-5 seconds without resorting to counting), and flexibility.*
2. Count within 1,000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting starting at any number. (2.NBT.2)
3. Place, order, and compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (2.NBT.4)
4. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (2.NBT.5)

[6] Explanations may be supported by drawings or objects.

Grade 3 Content Standards

Operations and Algebraic Thinking 3.OA

A. Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in five groups of seven objects each.
   For example, describe a context in which a total number of objects can be expressed as 5 × 7.
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
   For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[7]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
   For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.

B. Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations to multiply.[8]
   For example: When multiplying numbers order does not matter. If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (Commutative property of multiplication); The product 3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30 (Associative property of multiplication); When multiplying two numbers either number can be decomposed and multiplied; one can find 8 x 7 by knowing that 7 = 5 + 2 and that 8 × 5 = 40 and 8 × 2 = 16, resulting in 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive property); When a number is multiplied by 1 the result is the same number (Identity property of 1 for multiplication).
6. Understand division as an unknown-factor problem.
   For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

C. Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.
   For example, the product 4 x 7 = 28 has related division facts 28 ÷ 7 = 4 and 28 ÷ 4 = 7.
   a. MASTERY STANDARD-- know from memory all products of two single-digit numbers and related division facts.

D. Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations for problems posed with whole numbers and having whole number answers. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.[9]

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[7] See Glossary, Table 2.
[8] Students need not use formal terms for these properties. Students are not expected to use distributive notation.
[9] Students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.

For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Number and Operations in Base Ten 3.NBT

A. Use place value understanding and properties of operations to perform multi-digit arithmetic. [10]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. MASTERY STANDARD--Fluently add and subtract within 1,000 using strategies based on place value, properties of operations, the relationship between addition and subtraction and/or algorithms.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

Number and Operations—Fractions 3.NF

A. Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8.

1. Understand a fraction $^1/_b$ as the quantity formed by 1 part when a whole (a single unit) is partitioned into *b* equal parts; understand a fraction $^a/_b$ as the quantity formed by *a* parts of size $^1/_b$.

   Clarification: This benchmark emphasizes conceptual understanding through the use of manipulatives or visual models.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a unit fraction, $^1/_b$, on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $^1/_b$ and that the fraction $^1/_b$ is located $^1/_b$ of a whole unit from 0 on the number line.
   b. Represent a fraction $^a/_b$ on a number line diagram by marking off *a* lengths $^1/_b$ from 0. Recognize that the resulting interval has size $^a/_b$ and that its endpoint locates the number $^a/_b$ on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., $^1/_2 = {}^2/_4$, $^4/_6 = {}^2/_3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

   For example, express 3 in the form $3 = {}^3/_1$; recognize that $^6/_1 = 6$; locate $^4/_4$ and 1 at the same point of a number line diagram.
   d. MASTERY STANDARD--Compare and order fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

[10] A range of algorithms may be used.

Mastery Standards 3.MS

1. Know from memory all products of two single-digit numbers and related division facts (3.OA.7a) *Clarification: Students are fluent when they display accuracy (correct answer), efficiency (in about 3-5 seconds without resorting to counting), and flexibility.*
2. Fluently add and subtract within 1,000 using strategies based on place value, properties of operations, the relationship between addition and subtraction and/or algorithms. (3.NBT.2)
3. Compare and order fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (3.NF.3d)

Grade 4 Content Standards

Operations and Algebraic Thinking 4.OA

A. Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[11]
3. Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
    a. Know multiplication facts and related division facts through 12 x 12.

B. Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

C. Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
   For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Number and Operations in Base Ten 4.NBT

A. Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.

1. Recognize that in a multi-digit whole number, a digit in any place represents 10 times as much as it represents in the place to its right.
   For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

B. Use place value understanding and properties of operations to perform multi-digit arithmetic on whole numbers less than or equal to 1,000,000.

4. MASTRY STANDARD--Fluently add and subtract multi-digit whole numbers using the standard algorithm.

[11] See Glossary, Table 2.

5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions 4.NF

A. Extend understanding of fraction equivalence and ordering for fractions.

1. Explain why a fraction $^{a}/_{b}$ is equivalent to a fraction $^{(n \times a)}/_{(n \times b)}$ by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $^{1}/_{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions.

3. Understand a fraction $^{a}/_{b}$ with $a > 1$ as a sum of fractions $^{1}/_{b}$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (The whole can be a set of objects.)
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using drawings or visual fraction models. *Examples:* $^{3}/_{8} = {}^{1}/_{8} + {}^{1}/_{8} + {}^{1}/_{8}$; $^{3}/_{8} = {}^{1}/_{8} + {}^{2}/_{8}$; $2\ {}^{1}/_{8} = 1 + 1 + {}^{1}/_{8} = {}^{8}/_{8} + {}^{8}/_{8} + {}^{1}/_{8}$.
   c. MASTERY STANDARD--Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using drawings or visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction $^{a}/_{b}$ as a multiple of $^{1}/_{b}$.

   For example, use a visual fraction model to represent $^{5}/_{4}$ as the product $5 \times ({}^{1}/_{4})$, recording the conclusion by the equation $^{5}/_{4} = 5 \times ({}^{1}/_{4})$.

   b. Understand a multiple of $^{a}/_{b}$ as a multiple of $^{1}/_{b}$, and use this understanding to multiply a fraction by a whole number.

   For example, use a visual fraction model to express $3 \times ({}^{2}/_{5})$ as $6 \times ({}^{1}/_{5})$, recognizing this product as $^{6}/_{5}$. (In general, $n \times ({}^{a}/_{b}) = {}^{(n \times a)}/_{b}$.)

   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

For example, if each person at a party will eat $^{3}/_{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

C. Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[12]

For example, express $^{3}/_{10}$ as $^{30}/_{100}$, and add $^{3}/_{10} + {}^{4}/_{100} = {}^{34}/_{100}$.

6. Use decimal notation to represent fractions with denominators 10 or 100.

For example, rewrite 0.62 as $^{62}/_{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

7. MASTERY STANDARD--Order and compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

8. Identify the number that is one-tenth more, one-tenth less, one-hundredth more and one-hundredth less than a given number.

Example: One-hundredth less than 1.10 is 1.09.
Example: One-tenth more than 2.31 is 2.41.

Mastery Standards 4.MS

1. Fluently add and subtract multi-digit whole numbers using the standard algorithm. (4.NBT.4)

2. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction (4.NF.3c)

3. Order and compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (4.NF.7)

[12] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Grade 5 Content Standards

Operations and Algebraic Thinking 5.OA

A. Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols, e.g.,(6 x 30) + (6 x ½). Translate written real-world and mathematical descriptions into numerical expressions and numerical expressions into written mathematical descriptions.
2. *Example: The expression 4.5 + (3 × 2) in word form is four and five tenths plus the quantity 3 times 2.*
   Clarification: Expressions are limited to any combination of the arithmetic operations, including parentheses, with whole numbers, decimals and fractions.
3. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
   For example, express the calculation "Add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

B. Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
   For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Number and Operations in Base Ten 5.NBT

A. Understand the place value system.

1. Recognize that in a multi-digit number, including decimals, a digit in any place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. MASTERY STANDARD--Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,
      $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$.
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

B. Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. MASTERY STANDARD--Fluently multiply multi-digit whole numbers. (Include two-digit x four-digit numbers and, three-digit x three-digit numbers) using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the

relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and between multiplication and division; relate the strategy to a written method and explain the reasoning used.

Number and Operations—Fractions 5.NF

A. Use equivalent fractions as a strategy to add and subtract fractions.

1. MASTERY STANDARD--Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

   For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)

2. Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a set of objects), including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

   For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.

B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

   For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when three wholes are shared equally among four people each person has a share of size ¾. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

   a. Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

   For example, use a visual fraction model and/or area model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)

   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by:

   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

   For example, without multiplying tell which number is greater: 225 or ¾ x 225; $\frac{11}{50}$ or $\frac{3}{2} \times \frac{11}{50}$?

   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a

fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence
$^{a}/_{b} = (n \times a)/(n \times b)$ to the effect of multiplying $^{a}/_{b}$ by 1.

6. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[13]
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.

For example, create a story context for $(^{1}/_{3}) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(^{1}/_{3}) \div 4 = ^{1}/_{12}$ because $(^{1}/_{12}) \times 4 = ^{1}/_{3}$.

   b. Interpret division of a whole number by a unit fraction, and compute such quotients.

For example, create a story context for $4 \div (^{1}/_{5})$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (^{1}/_{5}) = 20$ because $20 \times (^{1}/_{5}) = 4$.

   c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

For example, how much chocolate will each person get if three people share ½ lb. of chocolate equally? How many $^{1}/_{3}$-cup servings are in two cups of raisins?

Mastery Standards 5.MS

1. Read, write, and compare decimals to thousandths. (5.NBT.3)
2. Fluently multiply multi-digit whole numbers. (Include two-digit x four-digit numbers and, three-digit x three-digit numbers) using the standard algorithm. (5.NBT.5)
3. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. (5.NF.1)

[13] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.